Fast Calculation of the Ballastless Track Impedance of High Speed Railway Using FEM and PEEC Modeling Approach

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The ballastless track impedance of high speed railway is important for designing the railway signal system. In order to consider the influence of the reinforcement bar, a two-step decomposition approach is proposed. The basic idea is evaluating the track impedance without the reinforcement bar using 2D finite element method (FEM), and the incremental impedance due to the reinforcement bar is calculated by the partial element equivalent circuit (PEEC) model. The numerical examples show that the proposed approach can guarantee the accuracy and reduce the computational time, at least 20 times, compared to using 3D FEM directly.

***Index Terms*—Equivalent circuits, Finite element analysis, Impedance, Rail transportation**

I. Introduction

In China, the high speed train, whose running speed is around 300 km/h, now has been put into business operation for years. The ballastless track, which can eliminate damages to the train coach by flying ballast at speeds in excess of 250 km/h, is widely used in practice. There are a bunch of reinforcement bars under the rail tracks. Apparently, these reinforcement bars will influence the series track impedance that is one of the most important parameters for designing the railway signal system. In this paper, an effort is made to calculate the ballastless track impedance efficiently.

II. Development of the Proposed Approach

Generally speaking, to calculate the track impedance, 3D model has to be considered because of the existence of the reinforcement bars under the tracks. If the finite element is adopted, the number of mesh becomes extremely big due to the skin effect of conductors and the small dimension of reinforcement bar, which results in a very high computational cost. In order to overcome this numerical obstacle, a divide and conquer strategy is proposed here. The total track impedance Z_{rail} can be decomposed to be:

$$Z_{\text{rail}} = Z_{\text{FEM}} + \Delta Z_{\text{PEEC}} \tag{1}$$

where Z_{FEM} is the track impedance without the reinforcement bar using 2D finite element method (FEM),and ΔZ_{PEEC} denotes the incremental impedance due to the reinforcement bar calculated using the partial element equivalent circuit (PEEC) model.

A. Calculation of Z_{FEM} Using 2D FEM

If the reinforcement bars are not included, the track impedance can be evaluated using 2D FEM, which is capable of modeling the geometries. In Fig.1, the governing equations with the magnetic vector potential (MVP) along the z-axis direction A_z as unknowns are:

$$\nabla(\frac{1}{\mu}\nabla A_z) - \mathrm{j}\omega A_z + \frac{1}{S}\iint_s \mathrm{j}\omega\sigma A_z \mathrm{d}s = -\frac{I_{\text{rail}}}{S},\ \text{in } \Omega_{1,2} \tag{2}$$

$$\nabla^2 A_z = 0,\ \text{in } \Omega_3 \tag{3}$$

$$A_z = \frac{\mu_0}{2\pi}\log(\frac{r''_{1\text{P}} r''_{2\text{P}}}{r_{1\text{P}} r_{2\text{P}}}),\ \text{on } \Gamma_{1,2}. \tag{4}$$

where S denotes the cross-section of the rail track 1 and 2; Ω_1, Ω_2 and Ω_3 are the region of rail track 1, 2 and non-rail track area; Γ_1 and Γ_2 are the boundary of the air area and the ground; σ and μ are the conductivity and the permeability of the rail tracks; The current flowing along the track, I_{rail}, is the total one;$r''_{1\text{P}}$, $r''_{2\text{P}}$, $r_{1\text{P}}$ and $r_{2\text{P}}$, respectively, represent the distance between a point P on the boundary and the ground image of the rail track 1, 2, the rail track 1 and 2. To reduce the FEM region, the boundary condition (4) is obtained using the complex depth concept [1] assuming the filament current flowing along the track.

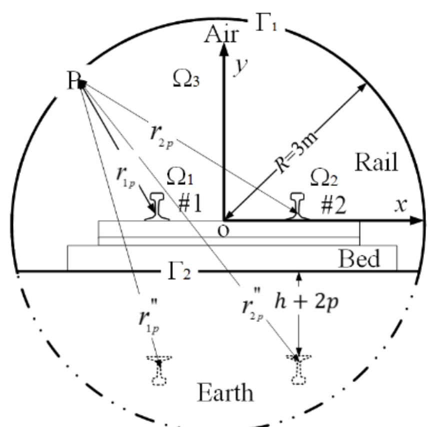


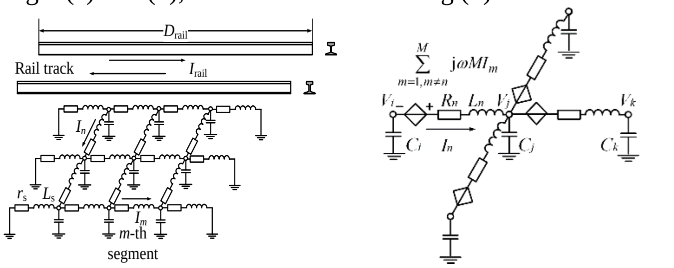
Fig. 1. Configuration of the rail track in 2D FEM model.

B. Calculation of the Incremental Impedance ΔZ_{PEEC} Using PEEC Modeling Approach

To handle the influence of the reinforcement bars, for simplicity, the bed of railway is omitted. The conductor of the reinforcement bar is divided into elementary segments where the unknown quantities are approximated as locally constant. For the n-th segment, an EFIE in frequency domain is formulated, and a Galerkin type weighting technique is utilized, one can have [2]:

$$\int_{l_n} \boldsymbol{E}^{\mathrm{inc}}(r_n,\omega)\cdot \mathrm{d}\boldsymbol{l}_n = \int_{l_n} \boldsymbol{J}(r_n,\omega)\cdot \mathrm{d}\boldsymbol{l}_n + \frac{\mathrm{j}\omega\mu_0}{4\pi}\sum_{m=1}^{M}\int_{l_n}\int_{l_m}\frac{\boldsymbol{J}(r_m,\omega)}{|r_n-r_m|}\mathrm{d}l_m\cdot \mathrm{d}\boldsymbol{l}_n + \frac{1}{4\pi\varepsilon_0}\sum_{m=1}^{M}\nabla\int_{l_n}\int_{l_m}\frac{\rho(r_m,\omega)}{|r_n-r_m|}\mathrm{d}l_m \mathrm{d}l_n \tag{5}$$

where E^{inc} is the incident field generated by the current I_{rail}; $\boldsymbol{J}$ and ρ, respectively, are the current density and the charge density of the m-th segment. M is the total number of segments. The PEEC technique interprets (5) in terms of circuit elements and an equivalent circuit model, as shown in Fig. 2(a) and (b), can be constructed using (5).



(a) (b)

Fig. 2. PEEC model of the reinforcement bars under the rail track.

After soving the circuits in Fig.2,the incremental impedance due to the reinforcement bars can be calculated as follow:

$$\Delta Z_{\mathrm{PEEC}} = \frac{1}{D_{\mathrm{rail}}}\left[-\sum_{m=1}^{M}(Z_{1m}-Z_{2m})\frac{I_m}{I_{\mathrm{rail}}}\right] \tag{6}$$

where I_m denotes the current of each bar; D_{rail} is the length of the rail track. Z_{1m} and Z_{2m} represent the mutual impedance between the rail track 1or 2 and the m-th bar;

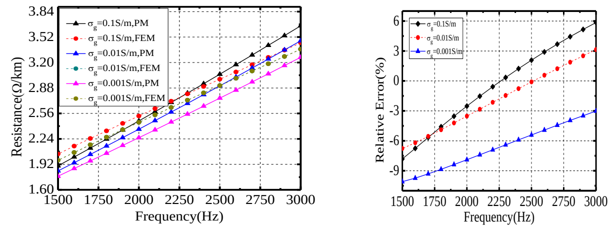
III. Numerical Examples

A. Validation of the Proposed Approach

In order to validate the proposed approach, the impedance of the ballastless rail track, whose reinforcement bar net, with the radius of 10mm,the depth of 100mm from the ground, is modeled as 250×200mm, will be calculated using both 3D FEM and the proposed method (PM). As σ_g=0.1, 0.01, 0.001S/m, the result is plotted inFigs.3 and4.From the figures, the reactance curves, using two methods, are almost overlapped. The resistance error is no more than 8% when σ_g=0.1S/m, and will get larger when σ_g=0.001S/m, but still in 10%. The comparison of the rail track impedance with the measurement result (ME), is listed in Tables I and II. In the proposed method, the earth conductivity σ_g=0.1S/m. The signal frequency varies from 1.7 to 2.6 kHz. From Tables I and II, the error for the resistance is less than 10%, and is no more than 7% for the reactance.

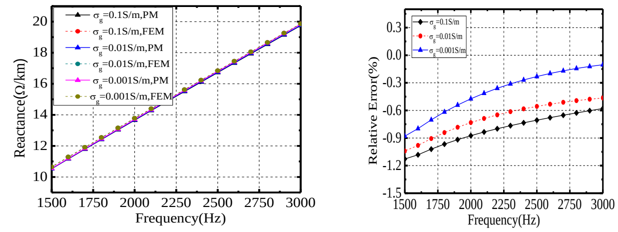
B. Comparison of the Computing Time

The most distinct merit of the proposed method is that the computing time is dramatically shortened, compared with the direct 3D FEM, the comparison of the computing time between the FEM and the proposed method at 2.6kHz, is listed in Table III, where the time saving ratio is defined as the computing time ratio using the PM and the 3D FEM. It can be found that the computing time is largely shortened from over ten minutes to less than one minute with the proposed method.



(a) (b)

Fig. 3. Resistance of the rail track with different earth conductivity.



(a) (b)

Fig. 4.Reactance of the rail track with different earth conductivity.

TABLE I

Comparison of Resistance Using Two Methods

Frequency /kHz	Resistance/Ω/km			Error /%	
	PM	3D FEM	ME	$\delta_{\mathrm{PM\text{-}ME}}$	$\delta_{\mathrm{3DFEM\text{-}ME}}$
1.7	2.13	2.24	2.38	-10.5	-5.88
2.0	2.46	2.53	2.61	-5.74	-3.06
2.3	2.82	2.81	2.85	-1.05	-1.4
2.6	3.17	3.08	3.08	2.92	0

TABLE II

Comparison of Reactance Using Two Methods

Frequency /kHz	Reactance /Ω/km			Error /%	
	PM	3D FEM	ME	$\delta_{\mathrm{PM\text{-}ME}}$	$\delta_{\mathrm{3DFEM\text{-}ME}}$
1.7	11.74	11.95	12.70	-7.54	-6.20
2.0	13.69	13.81	14.82	-7.65	-6.83
2.3	15.45	15.59	16.75	-7.74	-6.89
2.6	17.31	17.47	18.70	-7.82	-7.04

TABLE III

Comparison of Computing Time Using Two Methods

σ_g /S/m	PM /sec	3D FEM /sec	Time-saving ratio
0.1	30.5	882.8	28.9
0.01	41.8	852.9	20.4
0.001	44.1	1182.5	26.8

IV. Conclusion

In this paper, a fast approach to calculate the ballastless track impedance of high speed railway is proposed. The accuracy is comparable with the direct 3D FEM, and is also acceptable compared with the measurement result. The most considerable merit is that the computing time will be dramatically shortened to less than one minute with the proposed method.

V.References

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